

# Introduction

## *What I Learned in Elementary School*

*A poet must have his childhood  
close at hand.*

Theodore Roethke, Poet

A friend of mine left the hi-tech industry and a management career in mid-life and decided that his purpose in life is education. Not just any education, but mathematical education. In September 2000, just before the school year began, he called and told me — there's a project to promote mathematical education in Maalot, come join in. (Maalot is a development town in the far north of Israel. These towns were built during the 1950s, to settle new immigrants, and are usually considered to be rather backward.)

I teach mathematics at the university. Indeed, I have always been interested in teaching, and was involved in youth activities for many years. Among other projects, I used to instruct gifted elementary and high school students. However, I had not set foot in an elementary school since graduating the sixth grade myself. Therefore I consulted whomever I could. The advice I got was more or less unanimous: You have no idea what you're in for. Teaching gifted children is completely different from teaching ordinary children. Teaching in an elementary school is a profession. It is foolish to believe you can rely on your knowledge of the principles of teaching at the university. (At that time I believed that those, at least, I knew.)

I also discussed my dilemma with an experienced teacher whose opinion I valued. Upon hearing the idea, she burst out in a rage I never imagined her capable of. Don't you dare, she yelled at me. People like you are ruining elementary education. You will be no different from all the other academicians who have no idea how to teach in elementary school, and come to coach teachers with their fantasies, wreaking havoc in education. You'll go to Maalot, confuse everyone, and after they've been burned they'll be wary even of cold water for years.

Looking back, I can't believe I agreed in spite of all that. With the innocent conceit of a university professor, I assumed that I knew better than those who were, after all, only teachers. Looking back, I realize that heeding the advice I received would have cost me one of the most fascinating adventures of my life.

The slogan I embroidered on my banner was “hands on experience.” I would let the children experiment with mathematical concepts, and after experiencing them concretely, they would easily perform the abstraction. I started teaching in advanced classes — fourth and fifth grades. I took the children into the playground to measure the length of the shadows of trees, posts and buildings. We calculated the ratio between the children’s shadows and heights, and used this information to calculate the height of the trees according to their shadows. (This idea is borrowed from Thales, born in the 7th century B.C., the first mathematician in history to be mentioned by name, who used this method to calculate the height of the pyramids.) We drew circles on the pavement, measured radii, diameters and circumferences, and compared them. We measured the length and width of the classrooms in various ways. We learned how many floor tiles fit (lengthwise) in one meter, by computing the ratio between the length of the classroom in tiles and its length in meters.

I learned the price of conceit the hard way. There was very little meaningful teaching going on. Most lessons were a mess.

I remember my first day of insight well. I took a fourth grade class out to the playground, to measure the diameters and circumferences of circles drawn on the pavement. The teacher of the class was watching patiently the inevitable chaos that resulted, as the children took advantage of the opportunity to fool around. Eventually she suggested that we return to the classroom. In class we drew circles on the blackboard, and discussed the ratio between circumference and diameter. I was surprised to discover how easy it was to conduct an intelligent discussion with the children. I realized that I was underestimating the children’s power of abstraction. I also realized the power of words, and of interactive discussion.

Fortunately, I also began teaching first graders at that time. This was a thrilling experience. Meeting first graders is enlightening. They have not yet been corrupted; they trust you and will go along with you wherever you lead them. They respond directly, and immediately communicate what works for them and what doesn’t. There is no better place to learn how to teach than in the first grade. I also met an excellent teacher there, Marcel Granot, who was willing to join me in an adventure that enriched us both. I would open the lesson, and Marcel would intervene whenever she felt that the didactic aspects were less than perfect. This usually happened when I didn’t pace the lesson, that is, when I skipped a step.

From that day on, I have been learning, intensively and constantly, from each lesson and from every teacher I meet. The less successful lessons teach me as much as the successful ones. And the ones I learn from most are those that start out with a limp and then take off when the right thing is done.

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What have I learned? Much about teaching, about approaching children, about the way children think. I have learned about the importance of being systematic — something I desperately lacked at first. I understood that concepts adults perceive as a whole are actually built of many small elements, one upon the other, and that you cannot skip any one of them. I learned from personal experience that explanations are usually futile in elementary school: Concepts must originate in the child through personal experience. I was right about hands-on experience from the start — the problem was that I had no idea how to integrate it into the learning process. Hands-on experience need not necessarily apply to complex notions. It is essential also in acquiring the most basic concepts, such as the concept of the number, or what it means to be “greater than” or “less than.”

But aside from all that, I was in for a real surprise. Had I been told that in revisiting the elementary school I would myself learn mathematics, I would have never believed it. To my surprise, that is exactly what happened: I learned a lot of mathematics. Perhaps even mainly mathematics. Had I gone to teach in high school, this would probably not have been the case. The professional mathematician is familiar with the mathematics studied there, but in elementary school he is in for some novelties. It is there that the most basic elements arise: the concept of the number, the meaning of arithmetical operations. These are elements that the professional mathematician rarely pauses to consider.

A large part of what I learned wasn't new facts, but something completely different: subtleties. It was like looking at a piece of cloth — from afar it seems smooth and uniform, but up close you discover that it is made of fine, interwoven threads. What I believed to be one piece turned out to consist of a delicate texture of ideas. More importantly, I realized that to be a good teacher one must be familiar with the fine elements and the order by which they are interwoven. “Pacing, pacing, pacing,” as Marcel used to remind me.

This book is, to a great extent, about the subtleties that are at the base of mathematics, the subtleties that make it beautiful and provide meaning to its teaching.