

## Beauty

Poetry is always a search for the truth.  
(Franz Kafka, 1880-1924)

The American poetess Emily Dickinson was born in 1830 in Amherst, Massachusetts. She died in 1886 in the house in which she was born, and which she left only rarely, almost never in the last two decades of her life. Only seven of the 1,800 poems she wrote were published during her lifetime. And yet, she lived a full and rich life through her poetry, and brought much beauty into the world. The following poem is one of my favorites:

Adrift! A little boat adrift!  
And night is coming down!  
Will no one guide a little boat  
Unto the nearest town?

So Sailors say - on yesterday -  
Just as the dusk was brown  
One little boat gave up its strife  
And gurgled down and down.

So angels say - on yesterday -  
Just as the dawn was red  
One little boat - o'erspent with gales -  
Retrimmed its masts - redecked its sails -  
And shot - exultant one!  
(Emily Dickinson, "Poem # 30")

The poem touches the reader mainly through its sincerity: Dickinson is obviously speaking of herself. Through the metaphor of the boat she lays herself bare, to others and to herself, in a manner otherwise barred to her. "Adrift," "gave up its strife," "o'erspent with gales" – it is hardly likely that she would dare use these words to portray herself in real life. But the true power of the theorem is in the message that what is visible on the surface is less important than the inner forces. The little boat overwhelmed by tempest conceals a very brave vessel. On the surface of things, the boat is about to sink; in another dimension it spreads its sails and flies. Could Dickinson's life be depicted more beautifully than this?

Dickinson corresponded with a literary editor named Thomas Higginson, whom she admired, and who advised her against publishing her poems because they were unsuitable (they broke the conventional rules of rhyme and meter of the time; see, for example, the last stanza of the poem, that has five lines, in contrast with the four in each of the preceding stanzas). In one of her letters to him, she addressed the question that has always occupied scholars of poetry: What is a poem? She declared:

If I read a book and it makes my entire body so cold no fire can ever warm me, I know that is poetry. If I feel physically as if the top of my head were taken off, I know that is poetry. These are the only ways I know it. Is there any other way?

More than an attempted definition, this is defiance. Many poets protest the efforts by academics to pin down their artistic butterfly with definition. The English poet Alfred E.

Housman (1859-1936) replied to a request for such a definition: "I could no more define poetry than a terrier can define a rat, but [...] I thought we both recognised the object by the symptoms which it provokes in us" (Alfred E. Housman, *The Name and Nature of Poetry*).

Arthur Koestler (1905-1983) argued in a book entitled *The Act of Creation* (to which we will return later) that "laughter is the only physical reaction to abstract ideas." According to Dickinson, this is not accurate. Sometimes a poem, too, triggers a physical effect: it can bring about a shiver. Housman wrote in a similar spirit. He related that he knew that a line of poetry strayed into his thoughts if, when he was shaving, his skin bristled as if from cold. It is not by chance that Housman and Dickinson speak of "shivering." "Chilling" is perhaps the greatest compliment we can pay a poem. We feel like shivering from a feather's touch, and reading a poem is indeed like being touched by a feather: we aren't sure if it touched us or not. Reading about a boat adrift, we are not certain if the poem is really about a small, overwhelmed boat - or the poet's fate. We don't know if the Lea Goldberg's birds are there to exemplify the transparency of her windows-poems, or to tell us of the pain of dead loves.

Dickinson's definition does not relate only to the question of "What is poetry?", it also touches upon the question: "What is beauty?" Regarding the latter, mathematics, the uninvited guest, has the advantage: here there is near-unanimous consensus on the answer. Almost all mathematicians agree that a mathematical idea is beautiful if it reveals unexpected regularity. A beautiful idea should come, as if from nowhere, to clarify things as if by a wave of a magic wand. "Magic" is indeed a key word. Displacement, for example, is merely a diversion of attention, the trick of the magician who tells his audience: "Look what I am doing with my left hand" - while he performs the sleight of hand with his right. A metaphor, too, deceives the reader, because a different meaning sneaks in underneath the symbol on the surface. Unexpected ideas, that are so typical of poetry and mathematics, are yet another act of magic. The difference between the performing magician and the magicians of poetry and mathematics is that the latter are as ignorant as their audience regarding where the idea came from. Even the poet, or the mathematician, feel that the idea appeared from nowhere.

So, the sense of beauty is born of the unconscious grasp of a notion. The idea touches us in a deep place without our knowing how it does so. We experience beauty when we absorb a strong message, whether emotional or intellectual, on one level, while, on another, it is not fully understood.



Emily Dickinson , 1830-1886

## **Part I: Order**

God is a mathematician of a very high order.  
(James Jeans, English physicist, 1877-1946)

## The Strange Case of the Ants on the Pole

Only about myself I know how to speak.  
My world is as narrow as that of an ant.  
(Rachel [Blobstein], poetess, 1890-1931)

Some number of ants are on a one-yard long pole. The ants move - some to the right, others to the left, but all at the same speed: exactly one yard a minute. The pole is narrow, about as wide as a single ant, and when two ants meet they cannot continue on their way. At this point they behave like colliding billiard balls, that is, each changes direction and continues in the opposite direction, at the same speed.



When two ants meet (left) they change direction (right).

Every once in a while, an ant comes to the end of the pole, and then it falls and disappears forever.

In the end, will all the ants fall off the pole? And if so, how long will this take?

At first glance, we would think that this depends on the initial state, that is, on the number of ants on the pole and on their position. If there are many ants, it seems that it might take a long time for all of them to fall off. How can we test this? We have already discovered the first secret of mathematicians: looking at examples. All mathematical thought is conducted as a game between examples and abstractions. The difference between strokes in the two directions is that those in the direction of the concrete can be done consciously, that is, examples can be evoked deliberately. This is one reason to begin with examples, the other being that examples are the raw material of abstraction. In the case of the ants, the simplest example is that of a single ant. If the ant is at one end of the pole and advances to the other end, it will fall off in one minute. In any other instance, it will fall in less than a minute. But we have still not touched upon the core of the problem, since there was no collision in this example. So, let's look at two ants, and begin with a case in which, so it seems at least, it will take them longest to fall: two ants are located at the opposite ends of the pole, and they advance toward each other.



After half a minute, they will meet in the middle of the pole, they will reverse direction, and in an additional half a minute each will fall from the same end at which it started. If so, then both will fall after exactly one minute.

The next example is a bit more complicated. Ant A is at the right end, ant B is exactly at the center of the pole, and they are advancing toward each other.



They will meet at a distance of a quarter of a yard from the right end, after which ant A will go another quarter of a yard to the right before it falls off the right end, and ant B, that already went a quarter of a yard, will go left for  $\frac{3}{4}$  of a yard until it falls off the left end. All in all, ant B will cover a distance of one yard, and since it traverses a yard per minute, this will take it one minute.

This starts to look strange. In all three examples (one ant, two ants starting at different ends, two ants one of which begins in the middle), all the ants had fallen from the pole after one minute. Let's go one level higher of complexity, and examine three ants. Let's take the case in which ant A begins at the right end and moves to the left; ant B is at the left end, and advances to the right; and ant C is exactly in the middle, and moves to the right.



After a quarter of a minute ants A and C will meet and reverse direction. At the time of their collision, ant C is three-quarters of a yard from the left end, while ant B is a quarter-yard distant from the left end. This is how they appear:



After a quarter of a minute

After it changes direction, A will quickly fall from the right end. B and C are proceeding toward each other, and therefore they will meet in the middle of the pole. Until B and C meet, each will have moved for half a minute. When they meet now, they will reverse directions, and after an additional half a minute both will fall off. Once again - everything takes just one minute! Within exactly one minute, not a single ant will remain on the pole.

Now this is really strange. In all our examples, all the ants fell within a minute. Is there a general rule here? Does this always hold true? The answer is "yes," and the proof is not complicated, but it requires insight. suddenly revelation that makes everything simple. As often happens with abstractions, this insight does not add information, but ignores information: it ignores the identity of the ants. If we don't care who the ants are, then what happens at the moment that two ants meet? Actually, nothing happens. Before their meeting, one ant goes to the left, and the other to the right; after their encounter, the exact same thing happens: then, too, one ant proceeds to the left, and the other to the right, at the same speeds. But for our purposes, which ant goes to the left and which goes to the right doesn't matter.

The conclusion to be drawn is that we can completely disregard the collisions. All they do is confuse. The problem is completely identical to the problem: "Ants are proceeding along a pole one yard long, each at the speed of one yard per minute, without meeting and without changing direction. How long will it take for them to fall?" There's no mystery in this formulation. Each ant will fall off in a minute or less, depending on its initial distance from the end to which it heads.

Mathematicians are a lucky breed. They get paid to play. When we take into account the

billions that are invested in mathematical research and education, we could expect them to be occupied with practical subjects, but the reality is the exact opposite. They allow themselves to play with problems like the one with the ants. Why? Because the secret of mathematics' exceptional practicality can already be seen in this riddle. It reflects the discipline's primary strength: abstraction. This is expressed, before everything, in the presentation of the problem. The ants in the problem are mathematical: real ants don't move at a uniform speed, and they don't obey such simple rules, as do the ants in our problem. Mathematics is the study of systems that follow well-defined rules. But even more than in the presentation of the problem, abstraction is clearly visible in its solution. The secret lies in finding the inner rules, as if we revealed the hidden inner structure of the problem by use of X-rays

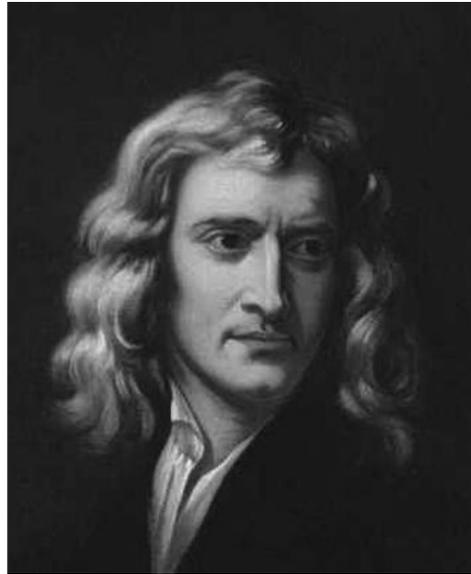
Ignoring the irrelevant, as in the ants problem, is a primary characteristic of mathematical thought. Mathematics takes the abstraction process to its extreme. It takes a tree that appears complex and intricate, strips it of its leaves, and reveals the trunk. Think, for example, of the concept of number. The person who invented the concept of "4" understood that, as far as the rules of arithmetic are concerned, it is immaterial if he had before him 4 stones or 4 pencils, and if pencils, what color they were, and how they were arranged. The concept of quantity is not dependent on such unimportant details..

Abstraction means discovering regularity, and regularity means generality. Generality saves thought effort: instead of finding out how many stones are 6 times 7 stones, and how many are 6 times 7 pencils, we only have to calculate the exercise  $6 \times 7$  once, and the answer will be valid for every type of object, anytime. The power of abstraction therefore consists of saving effort. "Mathematics is being lazy," said the mathematician George Polya (1887-1985), "mathematics is letting the principles do the work for you." In this respect, the ant question is very practical. Directly, it is not useful for anything, because there aren't any ants like these in the world, but it educates the person who solves it, or who learns the solution, to engage in abstract thought. It also sheds light on other problems in which similar principles will appear. And it is even possible that it was invented for the purposes of a real-world problem. Bundles of light waves ("solitons") behave in collisions just like the ants in the solution: they pass through one another. Physical phenomena often give birth to elegant mathematical problems.

## Hidden Order

Nature does nothing in vain, and more is vain, when less will serve; for Nature is pleased with simplicity.

(Isaac Newton)

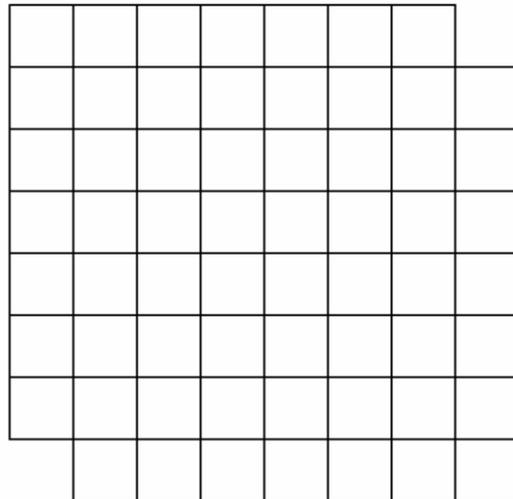


Isaac Newton, English mathematician and physicist (1642-1727). In 1666, his "miraculous year," he fled from the plague to the village where he had been born, where in one summer he developed the theory of gravity, several of the principles of modern optics, and differential and integral calculus. He spent his later years in disputes concerning his discoveries (especially with the German Gottfried Wilhelm Leibniz over the discovery of differential and integral calculus), in experiments in alchemy, and as master of the British royal mint

### The Power of Concepts

A good concept is like the key to a box that cannot be opened without it, or like a path that suddenly opens before you in a dark forest. A minute ago the thicket seemed impenetrable; but from the moment that the path was revealed, the way stands open. The English mathematician Andrew Wiles, who solved the famous Fermat's Conjecture, used another analogy. A good concept is like the switch of an electric light, that you find as you feel your way in a darkened castle. When you turn on the light, you know what's in the room where you are. Afterwards, when you go into the next room, you will have to feel your way again, looking for another switch.

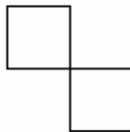
Here is a classic example, a problem that was composed in 1946 by Max Black, a British philosopher and mathematician. Take a board divided into 64 squares, that is, divided into 8 lengthwise, and into 8 breadthwise. Cut out the board's lower left-hand corner and its top right-hand corner, like this:



Can all 62 squares be covered by 31 dominoes?

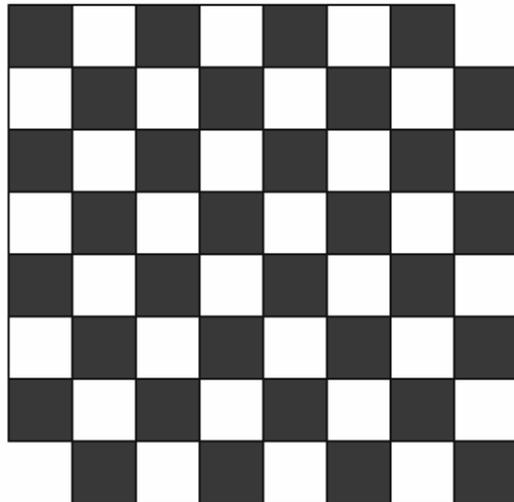
You also have 31 dominoes, each of which covers two squares on the board. All together, they would cover 62 squares, the same number remaining on the board now that the two corners have been cut off. Can the entire board be covered with these dominoes, in such a way that each domino covers two adjoining squares?

You may have guessed the first step in solving the problem: trying to solve smaller problems, and even very small ones. In this case, the smallest possible example is a board of  $2 \times 2$  squares. If we take a board of 4 squares, and remove the two opposing squares on the diagonal, we are left with:



This shape obviously cannot be covered with a single domino. The next case is a  $3 \times 3$  board, but this is clearly out of question: Removing 2 squares from a  $3 \times 3$  board leaves 7 squares, which is an odd number, and an odd number of squares cannot possibly be covered without overlapping, since each domino covers 2 squares. So, we should try a  $4 \times 4$  board. A short trial and error process will show you that in this case, too, the task is impossible.

In a small board checking all the possibilities is easy. Conducting such an assignment on an  $8 \times 8$  board is impractical, because there are too many possibilities to consider. What we need in this case is a concept that hits the mark. The appropriate concept in this case is the pattern of a chessboard:



Now everything becomes clear. Each of the 31 dominoes will cover one black square and one white square. Since the two squares we removed from the board are white, we are left with 32 black squares and only 30 white squares. These cannot be covered by 31 dominoes, that are supposed to cover 31 squares of each color.

The chessboard pattern revealed a hidden order. The colors of the squares were not there from the beginning, but as soon as they appeared, they simplified things as if by magic.

### The Chocolate Problem

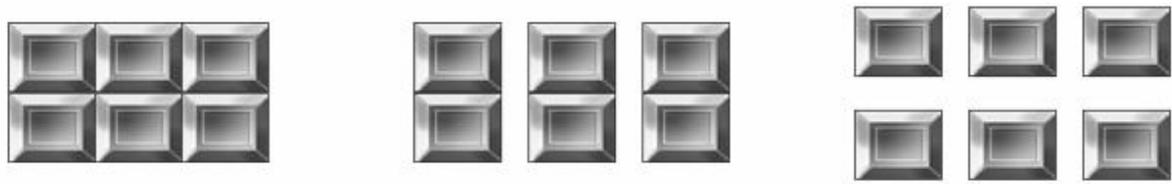
We have a chocolate bar measuring 5 blocks (or squares) long and 4 wide. We want to divide the 20 blocks among 20 children. In order to do this, we must break the bar, one step after another. In the first step, we break the bar along a single straight line, as we choose. In each step, we may take one of the pieces from the preceding step and break it along a single straight line. What strategy must we follow to use the smallest number of breakings?

We already know that it's best to begin with simpler examples, which in this case means having smaller dimensions. For example, a chocolate bar measuring 3 blocks in length by 1 wide, that is to be divided among 3 children. Here we have no choice: 2 breakings are necessary. Let's move on to a slightly bigger example: a bar measuring 3 in length and 2 wide, to be divided among 6 children. One way would be to first separate the two rows of 3 blocks apiece by a single breaking. After this, we need another 2 breakings for each of the 2 rows, for a total of 5 breakings.



One lengthwise breaking, and 2 more in each row, for a total of 5 breakings

Another way would be to separate the bar into 3 columns of 2 blocks apiece, by 2 breakings. In each of these 3 columns we need one additional breaking, for a total of  $2 + 3$  breakings. In this way, as well, we need 5 breakings.



Two vertical breakings, and 3 horizontal ones, for a total of 5 breakings

Examining larger examples, as well, will demonstrate the same simple rule: all the different ways lead to the same result.

**A chocolate bar of  $n$  blocks needs  $n-1$  breakings.**

The breaking strategy chosen has no effect on the number of breakings. We cannot separate the individual blocks with less than  $n-1$  breakings, nor with more than  $n-1$ . Why is this so? Here, again, the secret lies in finding the correct concept. This is **the number of pieces obtained after each breaking**. At the start the number of pieces is 1 - there is a single bar. Each breaking turns one piece into two, that is, it increases the number of pieces by 1. At the end of this process, there are  $n$  pieces. In order to go from 1 piece to  $n$  pieces, with each step adding a single piece, we need  $n-1$  steps. As usual, once we have found the correct concept, we can generalize. For example, the requirement for the breaking to be along a straight line is irrelevant. We can break any way we want, as long as each breaking turns a single piece into two pieces.